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On the Unique Solution of Planet and Star Parameters from an Extrasolar Planet Transit Light Curve

S. Seager

Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540 and The Carnegie Institution of Washington, Dept. of Terrestrial Magnetism, 5241 Broad Branch Rd. NW, Washington, DC 20015

G. Mallén-Ornelas

Princeton University Observatory, Peyton Hall, Princeton, NJ 08544 and P. Universidad Católica de Chile, Casilla 306, Santiago 22, Chile

Abstract. A unique analytical solution of planet and star parameters can be derived from an extrasolar planet transit light curve under a number of assumptions. This analytical solution can be used to choose the best planet transit candidates for radial velocity follow-up measurements. In practice, high photometric precision (< 0.005 mag) and high time sampling (< 5 minutes) are needed for this method. See Seager & Mallén-Ornelas (2003) for full details.

1. Assumptions

The following assumptions and conditions are necessary for a light curve to yield a unique solution of planet and star parameters:
• The planet orbit is circular (valid for tidally-circularized extrasolar planets);
• $M_p \ll M_*$ and the companion is dark compared to the central star;
• The stellar mass-radius relation is known;
• The light comes from a single star, rather than from two or more blended stars;
• The eclipses have flat bottoms. This implies that the companion is fully superimposed on the central star's disk and requires that the data are in a band pass where limb darkening is negligible;
• The period can be derived from the light curve (e.g., the two observed eclipses are consecutive).
In this article M is mass, R is radius, ρ is density, P is period, a is orbital semi-major axis, i is the orbital inclination, and G is the Gravitational constant. Where required the subscript p is for planet, $*$ for stellar, and $\odot$ for solar.

2. The Simplified Equations

Five equations are used to uniquely solve for M_*, R_*, a, i, and R_p. The simplified equations presented below require the additional assumption that $R_* \ll a$.

Transit depth

$$\Delta F \equiv \frac{F_{no\ transit} - F_{transit}}{F_{no\ transit}} = \left(\frac{R_p}{R_*}\right)^2. \tag{1}$$

Total transit duration

$$t_T = \frac{PR_*}{\pi a}\sqrt{\left(1+\frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}. \tag{2}$$

Transit shape (t_F = flat part of transit and t_T = total transit duration)

$$\left(\frac{t_F}{t_T}\right)^2 = \frac{\left(1-\frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}{\left(1+\frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}. \tag{3}$$

Kepler's Third Law

$$P^2 = \frac{4\pi^2 a^3}{GM_*}. \tag{4}$$

Stellar mass-radius relation

$$R_* = kM_*^x. \tag{5}$$

Here k is a constant coefficient for each stellar sequence (main sequence, giants, etc.) and x describes the power law of the sequence (e.g., $k = 1$ and $x \simeq 0.8$ for F–K main sequence stars (Cox 2000)). Note that Kepler's Third Law and the stellar mass-radius relation set a physical scale to two disks passing in front of each other. This breaks the geometrical degeneracy and allows a unique solution.

3. The Simplified Solution

The five parameters M_*, R_*, a, i, and R_p can be solved for uniquely from the above five equations. Moreover, the impact parameter $b \equiv a\cos i/R_*$ and stellar density ρ_* can be solved for uniquely without the stellar mass-radius relation.

$$b = \left[\frac{(1-\sqrt{\Delta F})^2 - \left(\frac{t_F}{t_T}\right)^2(1+\sqrt{\Delta F})^2}{1-\left(\frac{t_F}{t_T}\right)^2}\right]^{1/2}. \tag{6}$$

$$\rho_* = \frac{32}{G\pi}P\frac{\Delta F^{3/4}}{({t_T}^2 - t_F^2)^{3/2}}. \tag{7}$$

$$\frac{M_*}{M_\odot} = \left[k^3\frac{\rho_*}{\rho_\odot}\right]^{\frac{1}{1-3x}}. \tag{8}$$

$$\frac{R_*}{R_\odot} = k\left(\frac{M_*}{M_\odot}\right)^x = \left[k^{1/x}\frac{\rho_*}{\rho_\odot}\right]^{\frac{x}{(1-3x)}}. \tag{9}$$

$$a = \left[\frac{P^2 G M_*}{4\pi^2}\right]^{1/3}. \tag{10}$$

$$i = \cos^{-1}\left(b\frac{R_*}{a}\right). \tag{11}$$

$$\frac{R_p}{R_\odot} = \frac{R_*}{R_\odot}\sqrt{\Delta F} = \left[k^{1/x}\frac{\rho_*}{\rho_\odot}\right]^{\frac{x}{(1-3x)}}\sqrt{\Delta F}. \tag{12}$$

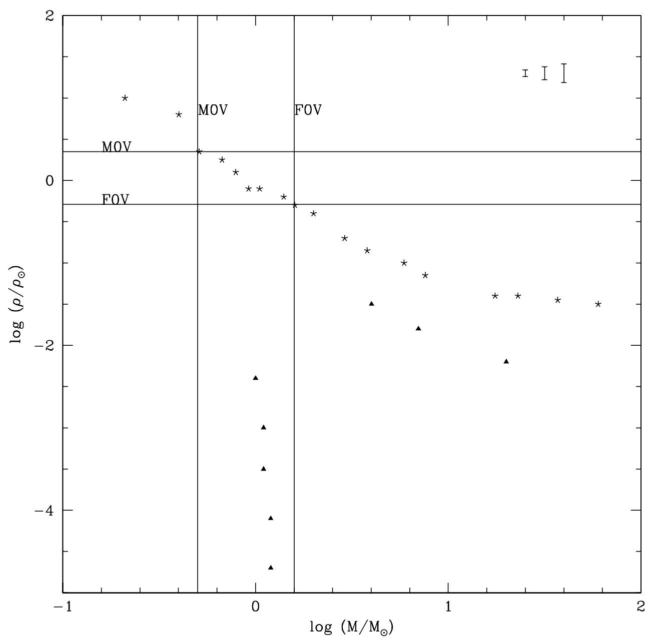

Figure 1. Stellar density ρ_* vs. stellar mass M_* (M_* is used as a proxy for stellar spectral type). See text for details. The box MOV to F0V shows the main sequence stars which are most appropriate for finding transiting planets. See Seager & Mallén-Ornelas (2003) for a discussion of errors.

4. Application

The above analytical solution has many applications, all related to selecting the best transit candidates for radial velocity mass follow-up. Here we only have room to describe one application; for others see Seager & Mallén-Ornelas (2003).

The stellar density ρ_* can be uniquely determined from the light curve alone without using the stellar mass-radius relation, as seen from equation (7).

A measured ρ_* can be used in three ways. (1) From the light curve alone a main sequence star and a giant star can be distinguished because main sequence stars occupy a unique position in a ρ_* vs. spectral type diagram (Figure 1). Hence a giant star with an eclipsing stellar companion can be ruled out. (2) From the light curve and the stellar mass-radius relation R_p can be estimated (equation (12)). Even for slightly evolved stars an upper limit on R_* and hence R_p can be derived. (3) A possibly common false positive planet transit can be ruled out by comparing ρ_* derived from the light curve with ρ_* derived from a spectral type. If the two ρ_* differ then something is amiss with the assumptions in Sec. 1. The possibly common case is the situation where a binary star system has its eclipse depth reduced to a planet-size eclipse due to the light from a third, contaminating, star (Figure 2). For a real example of this "blended star" situation, see Mallén-Ornelas et al. (2003).

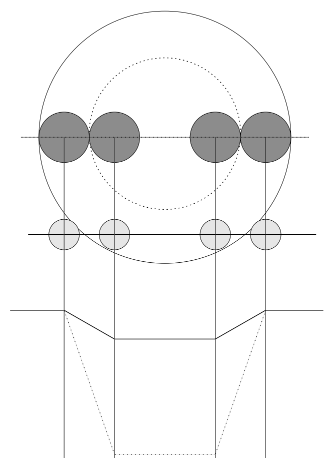

Figure 2. A deep binary star eclipse (dotted line) can mimic a planet transit (solid line) when extra light from a third, contaminating star (not shown) is present.

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References

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